Gaussianization: Enhancing the Statistical Power of the Power Spectrum

Mark C. Neyrinck

Abstract The power spectrum is widely used in astronomy, to analyze temporal or spatial structure. In cosmology, it is used to quantify large-scale structure (LSS) and the cosmic microwave background (CMB). This is because the power spectrum completely quantifies Gaussian random fields, which the CMB and LSS fields seem to be at early epochs. However, at late epochs and small scales, cosmological density fields become highly non-Gaussian. The power spectrum loses power to describe LSS and CMB fields on small scales, most obviously through high covariance in the power spectrum as a function of scale. Practically, this significantly degrades constraints that observations can place on cosmological parameters. However, if a nonlinear transformation that produces a (more) Gaussian 1-point distribution is applied to a field, the extra covariance in the field's power spectrum can be dramatically reduced. In the case of the roughly lognormal low-redshift matter density field, a log transform accomplishes this. Applying a log transform to the density field before measuring the power spectrum also tightens cosmological parameter constraints by a factor of several.

A Gaussian random field has convenient statistical properties. Its meaningful information is fully quantified by the power spectrum; all connected higher-order statistics vanish. Of particular importance for a measurer of (parameters which depend on) the power spectrum, all off-diagonal power-spectrum covariance matrix elements vanish for a Gaussian random field.

On small scales at late epochs, the cosmological overdensity field $\delta = \frac{\rho}{\langle\rho\rangle} - 1$ is highly non-Gaussian, with high off-diagonal power-spectrum covariance. However, this non-Gaussianity is not of an essential form, and can be largely removed with a monotonic transformation. The near-lognormality [1] of the density field may be exploited; a Gaussianizing transform like $\delta \to \ln(1+\delta)$ much reduces the power-spectrum covariance [2]. As shown in Fig. 2, it also reduces error bars on inferred parameters, reaching a factor of 5 reduction in the best case of the tilt n_s.

Mark C. Neyrinck
The Johns Hopkins University, Baltimore, MD 21218

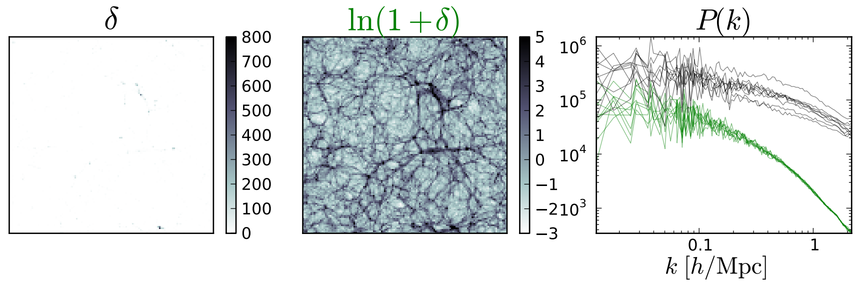

Fig. 1 Left: δ in a 2-h^{-1} Mpc slice of the 500-h^{-1} Mpc Millennium simulation (MS), viewed with an unfortunate linear color scale. Middle: the same slice with a logarithmic color scale. Right: the 2D power spectra P_δ and $P_{\ln(1+\delta)}$ of δ (black) and $\ln(1+\delta)$ (green), in 9 such slices. The wild, coherent fluctuations in P_δ illustrate its high (co)variance, absent in $P_{\ln(1+\delta)}$.

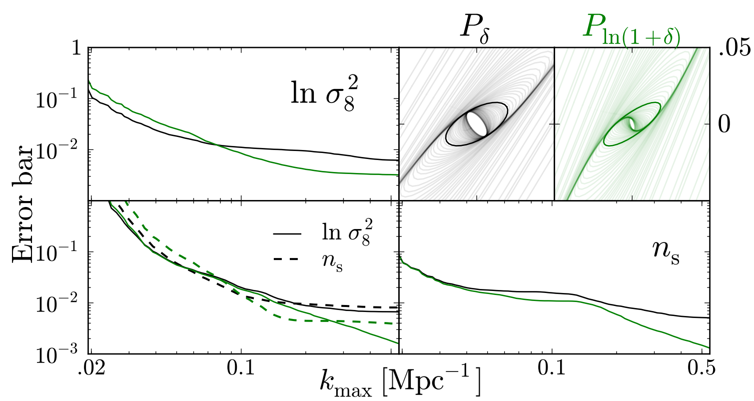

Fig. 2 Fisher-matrix estimates of error-bar (half-)widths and error ellipses for the cosmological parameters $\ln\sigma_8^2$ and n_s, inferred analyzing P_δ (black) and $P_{\ln(1+\delta)}$ (green). We show how they depend on the maximum wavenumber $k_{\max}$ included in a power-spectrum analysis of a 1-Gpc real-space matter density field. Diagonal panels show unmarginalized error bars over single parameters. In the lower-left panel, errors in each parameter are marginalized over the other. The upper-right panel shows how error ellipses contract as $k_{\max}$ increases. There is an ellipse shown for each $k_{\max}$ constituting the curves in the other panels. Outside the bold ellipses, analyzing only large scales where $k_{\max} < 0.1\,\mathrm{Mpc}^{-1}$, P_δ and $P_{\ln(1+\delta)}$ perform similarly. Inside the bold ellipses, nonlinear scales are included, up to the innermost ellipse that corresponds to $k_{\max} = 0.5\ \mathrm{Mpc}^{-1}$. Here, $P_{\ln(1+\delta)}$ greatly outperforms P_δ. See Ref. [3] for more details.

References

[1] P. Coles, B. Jones, MNRAS **248**, 1 (1991)
[2] M.C. Neyrinck, I. Szapudi, A.S. Szalay, ApJL **698**, L90, arXiv:0903.4693 (2009)
[3] M.C. Neyrinck, ApJ, submitted, arXiv:1105.2955 (2011)